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Considerations Regarding the Representation of Stock Levels Based on the Series of Periodic Functions

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Abstract

Current stocks vary between a minimum and a maximum value, according to a, usually linear, descending curve, depending on the economic process for which said stock is conceived. For a proper analysis of the development of stock levels throughout time, the information collected from the economic process must be transformed, so that mathematical analysis can be used both for building a model which simulates the development of the current stock, and for predicting the development of stock levels within the development of the economic process. Unlike the oversimplification provided by the theoretical material analyzing the stock process, in real life, the maximum and minimum stock levels are not constant and the periodicity of the characteristic stock events is different for every cycle of production or consumption. Using the series of periodic functions allows the opportunity to make use, for the analysis of the evolution of past, present and future stocks, of the well-known and established mathematical tools, for which programs of general use, that can be utilized both in academia and in the current economic practice, have already been designed.

Key words stock, supply, production cycle, Fourier series, periodic functions

JEL Codes: L11, L23, C30

Introduction

The shaping of stock levels, both in terms of volume and in terms of development throughout time, has imposed mathematical models able to simulate the necessity of a rational supply, with minimum expenditure of feedstock and materials in the production process, or of the finished product stocks and products for wide consumption in the activity of opening the goods. The current activity of the economic agents must make use of adequate functions for modeling the stocks, because if we see the stock as a supply of material, destined to satisfy the request of the beneficiaries, be they final clients (stock of finished products), be they representatives of a manufacturing service (feedstock or stocks of semi-finished products), be they a maintenance service (articles for current consumption or exchange parts), or a post-selling service (spare parts), the stock becomes an essential part of the trades made by the economic agent with the outside world; said trades must be controllable and predictable. As a direct consequence, the stock, no matter its nature, must also be controllable and predictable.

The essential trait of stocking processes is the accumulation of stock units, with the purpose of ensuring the availability of these units within a future economical process, meant to satisfy an external economic process, in harmony with one or more external partners. The modeling of the stocking processes must provide us with the scientific elements of analysis that can be used by the decision factors of the components of an economic process so that they can best approximate the costs associated with the stock variations over statistically significant periods from the past of the managed process, and realistically predict the future formation and stock exhaustion cycles.

The prediction of the stock level of an economic unit is interesting for the manufacturer as well as the consumer, because:

1) the manufacturer has a direct interest to maintain the stocks at medium levels that ensure optimal immobilization values, and straight from this, the producer will optimize the immobilization through stocks by reducing it to a minimum, calculated value;

2) the beneficiary wants his needs to be met instantaneously, this need of his directly contradicting the interests of the manufacturer, who has to satisfy the beneficiary's needs by avoiding, at a reasonable price, stock ruptures.

For the proper functioning of the economic process, a balance between the two aforementioned points of view must be reached, the purpose of the analysis and management of the stocking processes being identifying the solutions to finding this balance.

The analysis of any stocking process must be able to foresee the economic-financial circumstances of the factual development and the estimation of the conditions in which the stocking process will play its economic role.

The stock level and dynamics are determined by the market's development through the mechanism of external orders, the organizing and functioning of the stocking mechanisms being the object of the internal order and control processes of the economic agent. The maximum efficiency of the economic process can only be attained through the maximum utilization of the immobilized resources through the quantity of the stock and by ensuring the coordination of the processes joint to the stock development in a context favorable to the economic development pursued by the economic agent.

The modeling of the stocking processes through functions that ensure the effective analysis of past activity and fundamental predictions of future tendencies must, at the bare minimum, consider the following points:

- the provision of minimum stock levels, that should ensure the normal progression of the current activity of the economic agent through the fueling, according to necessity, of the consumption points, under the conditions of minimum expenses;
- the prevention of the accumulation of exaggerated stocks, with an insufficient turning rate;
- the provision of an optimal turning rate, to prevent the degradation of the stock elements;
- the possibility of operational use of the stock model within computerized management processes;
- applying managerial restrictions regarding the tracking and effective control of the stock.

1. Literature review

Numerous authors have conceived throughout time methods of analyzing stocking processes. A stock model contains mainly two categories of elements that work together towards shaping the image of the necessary resources for the economic agent's activity:

1. Elements pertaining to the economic agent's internal structure:

- The current stock (S_c) - the amount of resources necessary to ensure the continuity of the production process in-between two consecutive re-stockings, with the following parameters:
  - $I_1 \ldots I_n$ = the maximum level of the current stock;
  - $C_1 \ldots C_n$ = the minimum level of the current stock;
  - $T_1 \ldots T_n$ = the consumption period for the current stock.
- other types of stock depending on the economic agent's activity The safety stock (Ss) - the amount of resources necessary to ensure the continuity of the production process in the case of a halt in restocking, The preparation or conditioning stock (Scd) – the amount of resources necessary to ensure the continuity of the production process in the event of pre-production conditioning, The stock for internal transport (Sti), Intermediate stocks (Sin), Anticipation stocks (San), Seasonal stocks, circumstantial stocks (Scj), Strategic stocks (Ssg)

2. *Analysis elements* as functions, structures and algorithms, which turn the aforementioned notions into systems that can describe, analyze and predict the development of the aforementioned parameters and of the costs associated with an economic agent's stock management processes.

The economic literature has a number of authors who have developed both mathematical models and stock analysis methods, several stock functions having been created, and which can loosely be separated into two categories:

- functions based on the idea of cyclic stock, functions that take into account minimum and maximum stock level values which develop based on production cycles that allow periodic restocking, between fixed and equal time intervals;
- functions that are not based on the idea of a cycle, but which allow the possibility of both stock level variation and the variation of the exchange intervals with the external economic environment.

Whatever the representation of the stock function within the given time frame, between the maximum stock, I_k , and the minimum stock, C_k , the specialty literature allows for a linear variation of the stock level, implying the fact that between the maximum current stock level and the minimum current stock level, equal stock units are consumed at equal rates. The majority of the authors have defined for this type of relationship a stock function, defined across the chosen time intervals using an expression such as:

$$S_n(t) = \frac{C_{n+1}-I_n}{T_{n+1}-T_n} \cdot (t - T_n) + I_{n-1} \qquad (1)$$

where all the notations have been detailed above, *t* represents the current time, and T_n represents the restocking or opening time period.

This type of representation has both advantages and disadvantages, which are acknowledged and assumed by the stock managers in the economic process. Though it implies a certain consumption rate within the production process, which may or may not be real and specific to a certain economic agent, if significant deviations from the linearity of the consumption process are noticed, other functions, that can better estimate a non-linear consumption distribution, can be defined using the same structure, and this does not essentially involve any deviation from the classical theory of stock management, but only a temporary complication of the mathematical apparatus. From the economic agents' point of view, a major advantage of this type of linear representation is the simplicity with which mathematical manipulations can be executed, the swiftness with which quantitative approximations of stock levels, of different types of costs involved in the management process can be obtained, and the possibility to make individual use of the established and widely used ways to optimize stocks and costs associated with them. In the real economy, where economic agents can't always afford complex means of investigating the socio-economic phenomenon in which they participate, linear representations according to expression (1) describe the studied process very well, but only within a well-defined time frame and only up to a characteristic size, which depends on the measurement unit of the stock or the analyzed production.

This also shows us the main disadvantages of this type of over-simplified approach. Just like any over-simplification, it attracts a certain loss of data coherence between various time intervals, it doesn't support the creation of a complex economic model and it can't be used reliably for the analysis of past time intervals, let alone for making mathematical predictions using market development data, neither regarding production volumes, and, implicitly, stock volumes, nor regarding the evolution of costs and prices in the specified market segment.

The authors of this article positively appreciate the possibility and the utility of simplified calculations, and regard linear approximations such as expression (1) as the starting theoretical basis for any development of stock representation methods based on the current economic theory, according to the cited sources.

This article proposes the formulation of stock functions that can be used for complex processing, using means of analysis that have already been established in other fields, and for which there are tools designed by important institutions for analysis and process prediction, through numerical methods implemented for modern means of calculation. Two scenarios are analyzed: one in which the restocking is made with approximately equal quantities at equal time intervals (constant request at equal time intervals), and one in which neither the restocked quantity nor the time intervals at which the restocking takes place are constant (variable request at unequal time intervals).

2. Research methodology and data

Stock representation using series of period functions

As we have mentioned above, there are roughly two situations that correspond to the actions of restocking or opening in the economic agent's activity: the situation in which the economic agent restocks at equal time intervals and with (approximately) equal quantities of feedstock and semi-finished products, which roughly corresponds to a production and opening of the products on a market capable of sustaining this type of production, and the situation in which the economic agent restocks at unequal time intervals (irregular restockings) and with variable quantities of feedstock and semi-finished products, which roughly corresponds to a production and opening based on orders.

The analysis of the first situation, the rhythmical restocking one, is roughly the easiest one simulate, because, based on the expression (1), we must obtain the coefficients a_0, a_n and b_n of a Fourier series:

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n cos\ \frac{n\pi x}{L} + b_n sin\ \frac{n\pi x}{L}) \qquad (2)$$

A graphical representation of this function can be found in fig. 1.

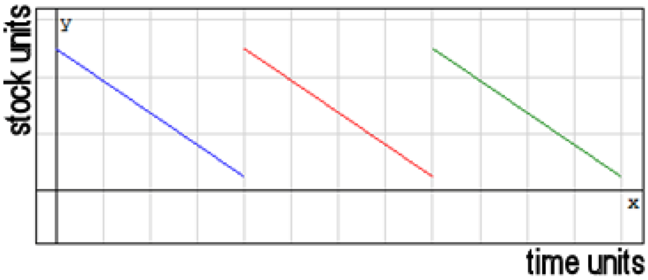

Figure no. 1. Graphical representation of periodic stocks
Source: Authors' processing

In order to generally find the aforementioned coefficients, calculations must be effected as follows:

$$a_0 = \frac{2}{T_{n+1}-T_n}\int_{T_n}^{T_{n+1}}[\frac{C_{n+1}-I_n}{T_{n+1}-T_n}\cdot(t-T_n)+I_{n-1}]dt = C_{n+1}-I_n \quad (3)$$

$$a_n = \frac{2}{T_{n+1}-T_n}\int_{T_n}^{T_{n+1}}[\frac{C_{n+1}-I_n}{T_{n+1}-T_n}\cdot(t-T_n)+I_{n-1}]\cdot cos(\frac{2n\pi t}{T_{n+1}-T_n})dt = 0 \quad (4)$$

$$b_n = \frac{2}{T_{n+1}-T_n}\int_{T_n}^{T_{n+1}}[\frac{C_{n+1}-I_n}{T_{n+1}-T_n}\cdot(t-T_n)+I_{n-1}]\cdot sin(\frac{2n\pi t}{T_{n+1}-T_n})dt = \frac{I_n-C_{n+1}+2I_{n-1}}{n\pi} \quad (5)$$

3. Results and discussions

For each of the aforementioned coefficients, we considered the difference $T_{n+1}-T_n$ to represent a restocking period, regardless of the amount of time this represents (1 day, 7 days, 1 month, 1 year).

By replacing the coefficients a_0, a_n and b_n in the expression (2), successive approximations of a stock function that corresponds to the enounced situation and the considered criteria can be obtained. Graphical representations of these successive approximations can be found in fig. 2.

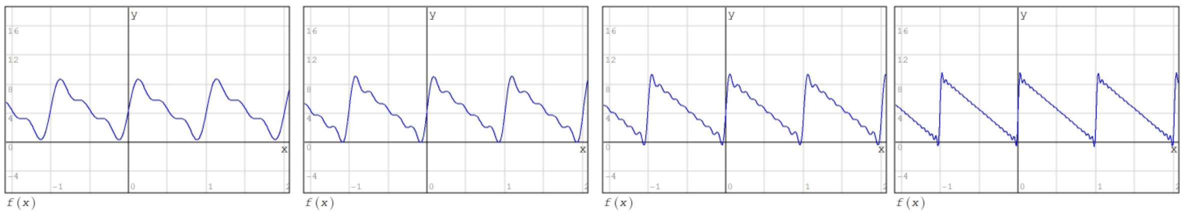

Figure no. 2. Graphical representation of the successive approximations of the function from expression (2)
Source: Authors' processing

The necessary and sufficient condition for setting the number *n* of terms in the series is that the difference between the linear function on intervals (expression (1)) and the representation of the same stock through periodic functions has to be less than the stock unit.

The analysis of the second situation, the one in which the restocking does not happen periodically, and in which the quantity of re-stocked products is not the same for each restocking, involves two hypotheses that are necessary for coordinating the numerical reality of the physical stocks and the mathematical rigor of the imposed conditions. The first hypothesis is that we can't find a general function, linear in its nature, that describes at any given time the physical state of the stock, and for this reason we will use numerical values of the state of the stock at various times, numerical values that we assume we can obtain at any given time by simply measuring the existing stock, so that we can obtain a stock sampling rate, satisfying in terms of Nyquist's sampling theorem, for each time interval that can be deemed significant to the stock's development. In practice, for a better mathematical representation of the stock function, much higher sampling rates are recommended. The second hypothesis is that any stock process that we store can be imaginarily extended back and forth through time, so that we can form a virtual, periodic restocking process, useful to the mathematical modeling and with no impact on the real stocking process.

Using the equation (1), we have determined for this article samples of stock represented in fig. 3.

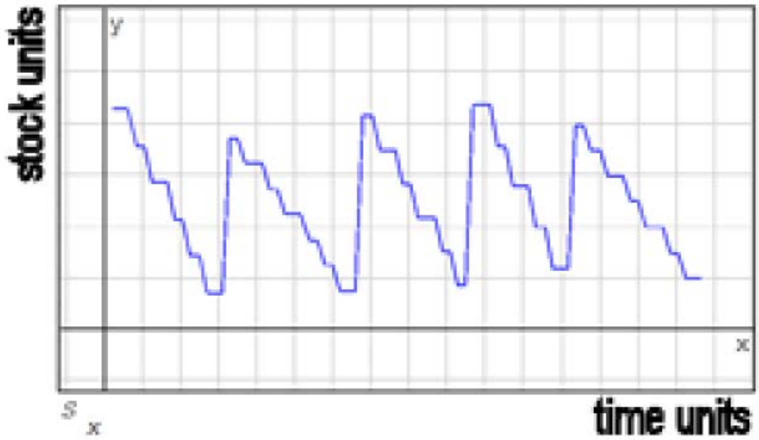

Figure no. 3. Stock samples
Source: Authors' processing

Using the notations described at the beginning of the article, we have effected a numerical Fourier transformation, whose algorithm can be found below. We mention that the algorithm presented below is not, in any way, neither elegant nor efficient; much better implementations are found both in the public domain and as commercial solutions. This article targets the simplicity of display and the ease of understanding, and does not focus on offering out-of-the-box solutions.

```
for(k=1, k≤N/2, k=k+1)                                          (6)
        Re_k=0
        for(n=1, n≤N, n=n+1)
                Re_k=Re_k+S_n.cos(2.π.n.k/N)
        Im_k=0
        for(n=1, n≤N, n=n+1)
                Im_k=Im_k+S_n.sin(2.π.n.k/N)
```

Following the algorithm, a normalisation would be necessary:

```
for(k=1, k≤N/2, k=k+1)                                          (7)
                Re_k=Re_k/(N/2)
                Im_k=-1.Im_k/(N/2)
```

With special values for Re_1 and $Re_{N/2}$: $Re_1=Re_1/2$, and respectively: $Re_{N/2}=Re_{N/2}/2$.

With the obtained data we can calculate a matrix of the system's amplitudes and phases:

for(n=1, n≤N/2, n=n+1) (8)

$AMFA_{n,1}=\sqrt{Re_n^2+Im_n^2}$

$AMFA_{n,2}=arctan(-\frac{Im_n}{Re_n})$

In order to obtain a continuous and differentiable representation of the stock function, there are, at this moment, two possibilities: either recompose the function as a function of time, from the system's amplitude and phase, obtained above, or an inverse Fourier transformation, obtaining a function of time as well. The two approaches are similar in terms of results and utility, but in order to implement them successfully, in each of both cases, but especially in the first one, we must apply filters that eliminate unwanted mathematical residues and which correct the mathematical functions' implementation domain in the case that we use commercial programs for processing. If we want the reconstruction of the function from phase and amplitude we use:

$$f(t)=\sum_{i=1}^{N/2} AMFA_{i,1}\cdot cos(\frac{2\cdot\pi\cdot i}{N}t+AMFA_{i,2}) \quad (9)$$

A graphical representation of the stock function, using the expression above, can be found in fig. 4.

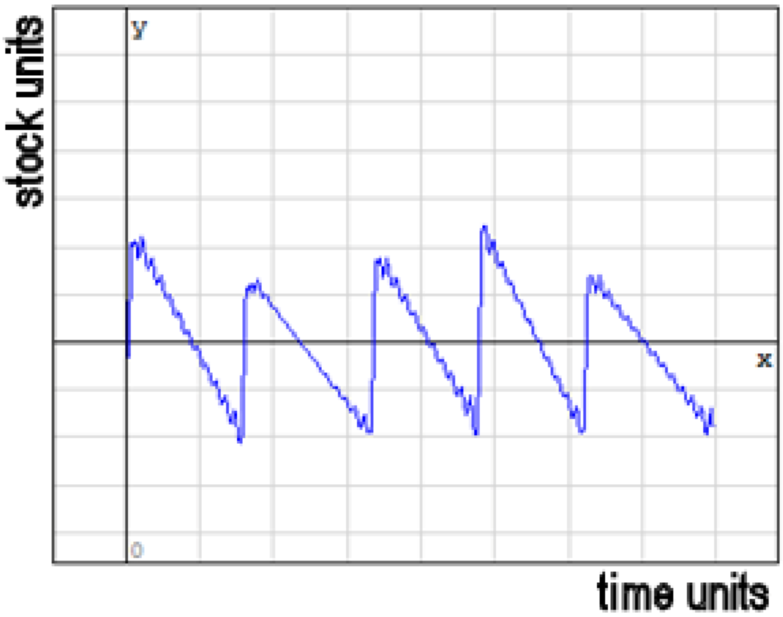

Figure no. 4. Graphical representation of the function obtained from the expression 9
Source: Authors' processing

If we want to compute the function using the inverse Fourier transformation, we use the following expression:

$$f(t) = \sum_{i=1}^{N/2} Re_i \cdot cos\,(\frac{2\cdot\pi\cdot i}{N}t)\ + Im_i \cdot sin\,(\frac{2\cdot\pi\cdot i}{N}t) \qquad (10)$$

A graphical representation of the function for determining the stock using the expression above can be found in fig. 5.

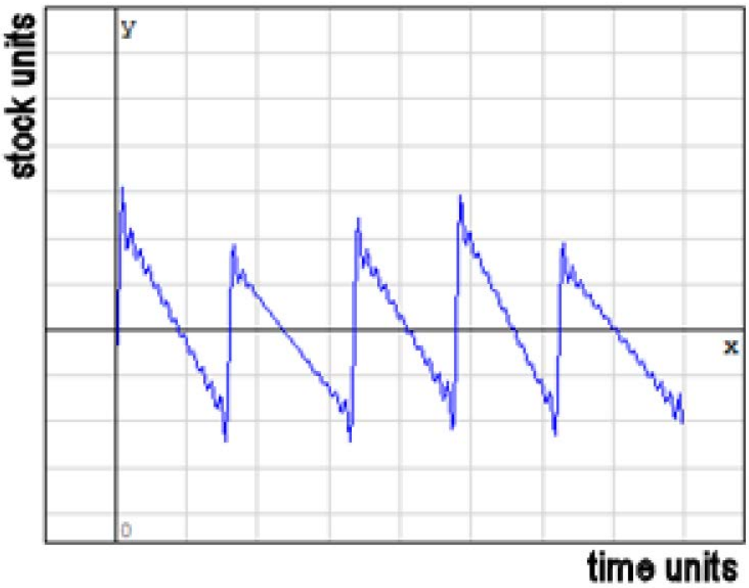

Figure no. 5. The graphical representation of the function obtained from the expression 10
Source: Authors' processing

Expressions (9) and (10) can be presented both as algorithms and as expressions. The authors of the article have preferred the expression form thanks to the clarity of the display.

In this scenario as well, the necessary and sufficient condition for setting the number n of terms in the series is that the difference between the linear function on intervals (expression (1)) and the representation of the same stock through non-periodic functions (expressions (9) and (10)) has to be less than the stock unit.

Conclusions

The means of representing stock functions proposed in the current article has, like any other mathematical means of representation, advantages and disadvantages that can recommend the proposed method for use in some well-defined situations, and that can invalidate it as a useful and efficient method in other situations. To state the disadvantages of this method first, it is evident from the start that it involves a large quantity of numerical information that has to be manipulated and inserted into the mathematical equations by the operator, but, as an extenuating circumstance, in today's world there are specialized programs that need little to no human intervention at all for the analysis and the control of the studied process. This undoubtedly means that the proposed representation does not lend itself to be used for calculations effected by a human operator, not because that would be impossible, but because the time in which the mathematical calculations would be effected would certainly be too long; this time could be used by the human operator more efficiently in conjunction with a specialized computing program implemented on a modern computer system. On the other hand, the proposed representation method lends substantial advantages when it comes to analyzing stock developments over successive time intervals that are important to the duration of the production cycle, because it is a method that can make use of complex mathematical instruments, that aren't available in the classical methods of investigating stocks, their evolution, and the costs associated with them. This method also brings an expansion of the representations currently existent in the specialty literature and paves the way for the utilization of a mathematical representation that can lead to feasible predictions of necessary stock levels and of the time intervals at which it is necessary for them to be restocked, depending on the evolutions of certain market parameters that intervene in the economic process. Using the mathematical methods of investigating the potential proposed by the current article lends certain legitimacy to the economic decisions, which become grounded in a decisional computing process based on a rational representation of the restocking or consumption processes investigated or managed by the economic agents. This way of representing stock functions through the significant terms of the associated Fourier series opens the door wide for numerical manipulations resembling the harmonic analyses paired with very powerful instruments for isolating, filtering, and, depending on the case, processing the data that can be identified through certain patterns of relational behavior, deduced from the evolution of the stock management processes.

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